Format for submission of Ph.D. Synopsis / Work plan

Title

Development of stochastic modeling and simulation of earth system modeling

Abstract of proposed work plan/ problem:

The thermal structure of the Earth's crust is influenced by its geothermal parameters such as thermal conductivity, radiogenic heat sources and initial and boundary conditions. Basically two approaches of modeling are commonly used for the estimation of the subsurface temperature field i.e (1) deterministic approach and (2) the stochastic approach. In the deterministic approach the subsurface temperature field is obtained by assuming that the controlling thermal parameters are known with certainty. However, due to inhomogeneous nature of the Earth's interior some amount of uncertainty in the estimation of the geothermal parameters are bound to exist. Uncertainties in these parameters may arise from the inaccuracy of measurements or lack of information about the parameters themselves. In the stochastic approach the uncertainties in parameters are incorporated and an average picture of the thermal field along with its associated error bounds is determined. To assess the properties of the system at a glance we need to obtain the mean value that gives the average picture and the variance or the standard deviation that is the variability indicator that indicates the errors associated with the system behavior due to errors in the system input.

Delineation of the thermo-mechanical structure of the crust and lithosphere using available geophysical and geological data is of great importance. The stochastic heat conduction equation for 1D and 2 D models shall be solved for different sets of boundary conditions, an exponentially decreasing radiogenic heat generation and randomness in thermal conductivity. Closed form analytical expressions for mean and variance in the temperature depth distribution have been used and an automatic formulation has been developed in matlab for computing and plotting the thermal structure. The matlab toolbox presented allows us in giving the controlling thermal parameters on the screen directly and it displays the subsurface thermal structure along with its error bounds. The software developed can be used to quantify the thermal structure for any given region. The solutions obtained shall be applied to quantify the subsurface thermal state in different tectonic regions in the Southern Indian Shield. The results shall be integrated with the seismic tomography results.

1. Introduction

Precise knowledge of the subsurface thermal field is very essential in understanding the global processes. The thermal structure of the Earth's crust is influenced by its internal parameters such as thermal conductivity and radiogenic heat sources and external parameters such as surface temperature and basal heat flow. Due to the uncertainty inherent in the heat sources and thermal parameters the current emphasis is to extend the deterministic thermal model to the stochastic model. If we incorporate randomness in the heat source or thermal conductivity or the boundary conditions the governing differential equation becomes stochastic in nature. Uncertainties in these parameters may arise from the inaccuracy of measurements or lack of information available on them. Generally when a stochastic solution is sought one looks for the first and the second order statistics. These two moments are known to quantify the uncertainties in the system output due to the uncertainties in the system input.

Solution to transport equations under a stochastic framework can be obtained both analytically and numerically. Among them the Monte Carlo numerical simulation method and the small perturbation method which are used most widely. While the stochastic approach has provided some insight into the phenomenon of transport in the Earth, several shortfalls do exist. One such limitation is the

assumption of small randomness required in the perturbation approach. Also existing perturbation approaches can handle randomness in one quantity only. The stochastic governing flow equation is solved using a new approach to the perturbation method where the solution is built using a series expansion method. In this approach, the Adomian method, the assumption on the probability distribution is avoided instead of its information pertaining to the spatial correlation structure is used. This information on the spatial correlation structure on the data can be easily available from field measurements and hence has more scope from an applied point of view. The approach is also known to handle large variance in the controlling parameters.

2. Review of Literature and Development in the subject (Previous work done in the relevant area):

Quantifying the thermal structure helps in understanding the dynamical behavior of the continental lithosphere which are essential for understanding the evolution of the continents, nature of lithosphere/sub lithosphere mantle coupling, relation between surface features and deep seated processes. Hence, it is necessary for obtaining the thermal structure of the lithosphere through deterministic or the stochastic modeling. The controlling parameters such as the radiogenic heat production, thermal conductivity, initial and boundary conditions are assumed to be known with certainty to estimate the subsurface temperature field using deterministic approach. However, due to the compositional variations and anisotropic character of the Earth's interior large uncertainty is associated with measured or estimated values of the thermal conductivity, heat production or temperature. Poor knowledge about the parameters and insufficient sampling across the study area are the fundamental cause for the uncertainties. Hence, a deterministic thermal model with known/measured parameters is extended to the stochastic model by incorporating uncertainties in parameters and an average picture of the temperatures along with its associated error bounds is obtained. Existing methods for solving the stochastic differential equation is based on small perturbation method or the Monte Carlo simulation. These methods nevertheless have their own limitations. For a stochastic solution, one quantifies the mean, the variance while the standard deviation is used to quantify uncertainties in temperatures due to inherent uncertainties in the input thermal parameters or the heat sources.

3. Objectives of Research/ Proposed Hypothesis:

Quantification of accurate subsurface temperature and the thermal lithospheric thicknesses from the stochastic point of view.

To solve the stochastic heat conduction equation and obtain the error bounds on the 1D and 2D thermal state of the crust and the lithosphere: Application to Southern Indian shield.

4. Methodology to be adopted:

The stochastic thermal model has to be extended from the deterministic model due to the uncertainties in the heat sources and thermal parameters. Uncertainties in these parameters may arise from the inaccuracy of measurements or lack of information on them. By incorporating randomness in the heat source or thermal conductivity the governing equation becomes stochastic in nature and an average picture of the thermal field along with its associated error bounds is determined.

The 1-D steady state heat conduction can be described by

$$\frac{d}{dz}\left\{(\bar{K}+K'(z))\frac{dT}{dz}\right\} = A(z) \qquad (1)$$

$$\text{where } A(z) = -A_0 e^{-z/D} \qquad 0 \le z \le L$$

$$= 0 \qquad z \ge L$$

Where T is the temperature ($^{\circ}C$) , A(z) is the radiogenic heat source ($\mu W/m^3$) , K (z) is the thermal conductivity ($W/m\,^{\circ}C$), and z is the depth (m). The associated boundary conditions used are:

$$T(0) = T_0 \tag{2}$$

and the basal heat flow

$$\overline{K}\frac{dT}{dz} = Q_b \quad at \quad z = L \tag{3}$$

Where L is the depth of the model.

K(z) is the thermal conductivity ($W/m\,^{\circ}C$) which is expressed as a sum of a deterministic component and a random component where $\overline{K}$ is the mean value and $K'(z)$ is the random component with mean zero and a Gaussian colored noise correlation structure represented by

$$E(K'(z)) = 0 \tag{4}$$

$$E(K'(z_1)K'(z_2)) = \sigma_K^2 e^{-\rho|z_1 - z_2|} \tag{5}$$

where σ_K^2 is the variance in thermal conductivity, ρ is the correlation decay parameter (or $1/\rho$ is the correlation length scale) and z_1 and z_2 are the depths.

Here, the solutions to equation (1) with the associated boundary conditions (2) and (3) shall be obtained for two different cases i.e., for random radiogenic heat sources and random thermal conductivity. The heat source can be constant or an exponentially decreasing function. The solution to 2D thermal model with errors in controlling thermal parameters shall also be solved to obtain the error bounds on temperature field.

5. Importance of study/ Society application:

- International status

Exact stochastic differential and integral equations relating to diffusion problem has been used by Cheng & Lafe (1991) for ground water flow problems. Using yet another approach the decomposition method or the Neumann expansion method (Adomian, 1994; Serrano, 1995) the governing stochastic equation is solved. In heat flow studies Vasseur et al., (1985) quantified the uncertainty in the heat flow using the least squares inversion technique incorporating uncertainties in the temperature and thermal conductivities. In a later study using the probabilistic approach Vasseur& Singh (1986) and Nielsen (1987) quantified the effect of variation in heat source on the surface heat flow. In yet another approach, the random simulation numerical method, uncertainties in the thermal models are quantified due to uncertainties in the controlling thermal parameters (Royer & Danis, 1988; Gallagher et al., 1997). Using Markov Chain Monte Carlo method Ferrero &Gallagher (2002) quantified the uncertainties in the heat flow parameters.

Jokinen & Kukkonen (1999a, 1999b, 2000) used the random modeling technique for uncertainty analysis for forward geothermal modeling of the lithospheric thermal regime and is very useful in studying the nonlinear problems. Simple 1-D analytical solution to the mean behavior and its

associated error bounds is very useful in quantifying the uncertainty. For a purely conductive model 1-D approximation in steady state is justified since the width of the model generally exceeds several times its depth.

- National Status

Thermal modeling plays an important role in understanding the state of the lithosphere. For modeling the conductive heat transfer in the crust the important controlling parameters are the radiogenic heat production and the thermal conductivity. In case of a stabilized continental crust the conductive heat transfer in steady state condition is a reasonably good approximation. A package is developed to compute and plot the error bounds on the subsurface temperatures due to errors in the thermal conductivity for a 1-D steady state conductive earth model for surface temperature and basal heat flow as its boundary conditions.

The subsurface thermal parameters have been quantified in different regions in the Indian shield (Roy& Rao 1999, Roy & Rao 2003). In a recent study Pandey(2015) and Singh (2015) have brought out the application of the deterministic solution to the thermal state in the crust and lithosphere in different regions of the Indian shield. The 1-D steady-state stochastic heat conduction equation is solved using the perturbation technique incorporating a Gaussian correlation structure in the radiogenic heat sources (Srivastava & Singh, 1998). In a study using the decomposition method the stochastic heat equation is solved by Srivastava & Singh (1999) incorporating uncertainties in the thermal conductivity and temperatures and its associated error bounds obtained. Later Srivastava (2005) using the same decomposition method obtained closed form solution of the mean heat flow along with its error bounds. Srivastava et al., (2006) and Srivastava & Singh (2008) obtained the stochastic closed-form solution of the temperature and heat flow along with its error structure.

- Significance of the study

Closed form solution to the stochastic heat equation shall be obtained and error bounds on the thermal state will be quantified. These expressions can be used for any given region for quantifying the errors on the thermal structure at the base of the lithosphere and also give the lithospheric thickness for different regions. A Graphical user interface program shall be developed to compute the subsurface temperatures along with its error bounds. The 2_D thermal modeling shall be carried out and will be very useful in understanding the geodynamics of the study region. Quantification of accurate subsurface temperature, heat flow along with uncertainties will be carried out to decipher the thermal state of the crust in different tectonic regions of Indian shield and the lithospheric thicknesses. These studies will play a very important role in understanding the Geodynamics of the region.

6. Proposed work Plan/ Formulation and Structure of Study:

- **Year-wise Plan of work and targets to be achieved**

The Southern Indian shield is divided into three distinct segments i.e., Western Dharwar Craton (WBC), Eastern Dharwar Craton (EDC) and Southern Granulite Terrian (SGT). It is bounded by Godavari graben in the east and partly by Deccan basalts in the north. The Southern Indian shield is occupied by mid Archean and early Prolerozoic crystalline rocks. Th WDC is associated with mid Archean tonalitic crust where as the EDC consist of late Archean granodiortic crust. The SGT is late Archean gneiss-granulite crust. Earlier reaserches had suggested a thick lithophere but now with aquisition of large seismic data tomography researches have lowered the lithosphere thickness to

around 150 km. An unusual extremely thin, hot and deformed lithosphere for the Indian shield has been advocated by some researchers. After several years of theoretical and seismic/seismological and heat flow studies the nature of the Indian lithosphere being thick or thin still remains a question.

Subsurface thermal state with its uncertainties for different regions in the Southern Indian shield would be obtained and the results discussed in light of seismic tomography studies. A user friendly Graphical User Interface to compute and plot the thermal state in the crust and Lithosphere shall be developed.

Work plan

First Year	Solving stochastic differential equations incorporating Gaussian uncertainties to obtain the expressions for the subsurface temperatures and its error bounds Literature Survey
Second Year	Developing Matlab based GUI and solving the1 D and 2-D thermal state along with uncertainties : Application to the southern Indian shield Literature Survey
Third Year	Application to different tectonic regions in India. Writing and submission of the thesis.

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